Geometric View Factors for Parallel Directly Opposed Rectangles

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The mathematical expression for the radiative view factor (2) between directly opposed parallel rectangular surfaces shown in Figure 1 is

$$F = \frac{1}{\pi h w}\left[\ln \frac{(W^2+1)(H^2+1)}{(W^2+H^2+1)} - 2(W\tan^{-1} W + H\tan^{-1} H) + 2\left(H\sqrt{W^2+1}\tan^{-1}\frac{H}{\sqrt{W^2+1}} + W\sqrt{H^2+1}\tan^{-1}\frac{W}{\sqrt{H^2+1}}\right)\right] \quad (1)$$

Because this equation does not readily lend itself to hand calculations, values of the view factor have been calculated for specified values of the parameters H and W and published in graphical form (*1*). To obtain a view factor it is necessary to interpolate the graphical results with resultant inaccuracy. This leads one to desire an accurate, yet simple, expression for such determinations.

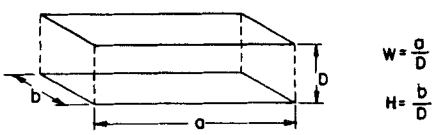

Fig. 1. Representation of directly opposed rectangular surfaces.

When applied to plane parallel, infinite length surfaces of equal width, Equation (1) reduces to

$$F = \frac{\sqrt{W^2+1}-1}{W} \quad (2)$$

For a surface with finite dimensions, Equation (2) yields a view factor which is too large since it does not delimit the length of the surface. However, by analogy with certain conduction heat transfer problems (*3*), the obvious step is to attempt a solution for the view factor for a surface with finite dimensions as the product of solutions for intersecting right angle surfaces of infinite length and finite width. Expressed in terms of the dimensionless parameters W and H this results in

$$F = \left[\frac{\sqrt{W^2+1}-1}{W}\right]\left[\frac{\sqrt{H^2+1}-1}{H}\right] \quad (3)$$

View factors computed by Equation (3) were generally found to be slightly low but began to agree quite well with the exact solution [Equation (1)] as H/W increased. To improve the agreement between the exact solution and Equation (3), an empirical correlation factor was determined. The resulting expression for the view factor is

$$F = \frac{\left(\frac{\sqrt{H^2+1}-1}{H}\right)\left(\frac{\sqrt{W^2+1}-1}{W}\right)}{\left(1-\frac{0.23}{\sqrt{H^2+W^2+1}}\right)} \quad (4)$$

This expression and the exact solution were programmed for digital computation to obtain an accurate comparison of the results determined by the simpler empirical equation. Computations of geometric view factors were performed for values of H/W of 1, 2, 10, 20, and 100 while H was varied over a range from 0.02 to 100. For all values of H/W, results from Equation (4) correlate with the results obtained from Equation (1) to within 1% for H ranging from 0.02 to 0.2, to within ½% for H ranging from 0.2 to 100, and exactly in the limits as H approaches zero or becomes infinite.

Equation (4) is significant in that it accurately relates the view factor, F, to the geometry and dimensions of the system by simple relationships and readily lends itself to hand calculations. Thus the engineer can more accurately determine view factors and radiant energy exchange between plane parallel rectangular surfaces.

NOTATION

a = surface dimension
b = surface dimension
D = normal distance between parallel surfaces
F = radiative view factor
H = b/D
W = a/D

LITERATURE CITED

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